A

Feb 6

I

II

III

IV

B

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 10 & 20 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 24 \\ 49 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{pmatrix} \overbrace{\boxed{\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}}^{\underline{z}} = \begin{pmatrix} 10 \\ 24 \\ 49 \end{pmatrix}$$

$$\underline{z} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 24 \\ 49 \end{pmatrix} \qquad z_1 = 10 \qquad z_2 = 4$$

$$2z_1 + z_2 = 24 \qquad 3z_1 + 4z_2 + z_3 = 49$$

$$20 + z_2 = 24 \qquad 30 + 16 + z_3 = 49$$

C $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \\ 3 \end{pmatrix}$

Backward $3x_3 = 3 \Rightarrow x_3 = 1$

$x_2 + 2x_3 = 4$

$x_2 + 2 = 4 \; ; \; x_2 = 2$

$x_1 + 2x_2 + 3x_3 = 10$

$x_1 + 2 \cdot 2 + 3 \cdot 1 = 10$

$x_1 + 7 = 10 \quad x_1 = 3$

$\underline{x} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

D) $\underline{\underline{A}}\,\underline{x} = \underline{b}$

LU Fac of $\underline{\underline{A}}$

$M_1 \to a_{11}$

$M_2 \to a_{22}$

$\vdots$

$M_{n-1} \to a_{n-1,n-1}$

$\underline{\underline{L}}\,\boxed{\underline{\underline{U}}\,x}^{\,z} = \underline{b}$

Solve $Lz = b$
Forward

Solve $Ux = z$
Backward

$M_{n-1} M_{n-2} \ldots M_3 M_2 M_1 A = U$

$\cancel{M_1 M_2 \ldots M_{N-1}}$

$M = M_{n-1} M_{n-2} \ldots M_2 M_1$

$MA = U$

$A = M^{-1} U = M_1^{-1} M_2^{-1} \ldots M_{n-1} \; U$

Ē

Norm of a vector

$$\underline{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

Integer $p = 1, 2 \ldots \infty$

$$\|x\|_p = \left(\sum_{k=1}^{n} |x_k|^p \right)^{1/p}$$

p-norm of x

$p = 1$ Manhaten

$p = 2$ Euclidean

$p = \infty$ infinity

F

l norm

$$\|x\|_1 = \sum_{r=1}^{n} |x_r|$$

$$\|x\|_2 = \left(\sum_{k=1}^{n} |x_k|^2 \right)^{1/2}$$

G Example $n = 2$

$$\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

$$\|\underline{x}\|_1 = |x| + |y| \qquad \|x\|_\infty = \max(|x|, |y|)$$

$$\|\underline{x}\|_2 = \sqrt{x^2 + y^2}$$

$$\|x\|_\infty = \lim_{p \to \infty} \left[|x|^p + |y|^p \right]^{1/p}$$

$$= \left[\begin{array}{ll} |x| > |y| & (a) \\ |x| < |y| & (b) \\ |x| = |y| & (c) \end{array} \right.$$

(a) $= \lim_{p \to \infty} \left(|x|^p \left[1 + \underbrace{\left(\frac{|y|}{|x|} \right)^p}_{\to 0} \right] \right)^{1/p} = |x|$

(b) $\lim_{p \to \infty} \left[|y|^p \left(\left(\frac{|x|}{|y|} \right)^p + 1 \right) \right]^{1/p} = |y|$

(c) $= \lim_{p \to \infty} \left[|x|^p \cdot (1+1) \right]^{1/p} = |x| \cdot 2^0$

H

$$\|\underline{x}\|_\infty = \max_{k=1..n} |x_k|$$

I

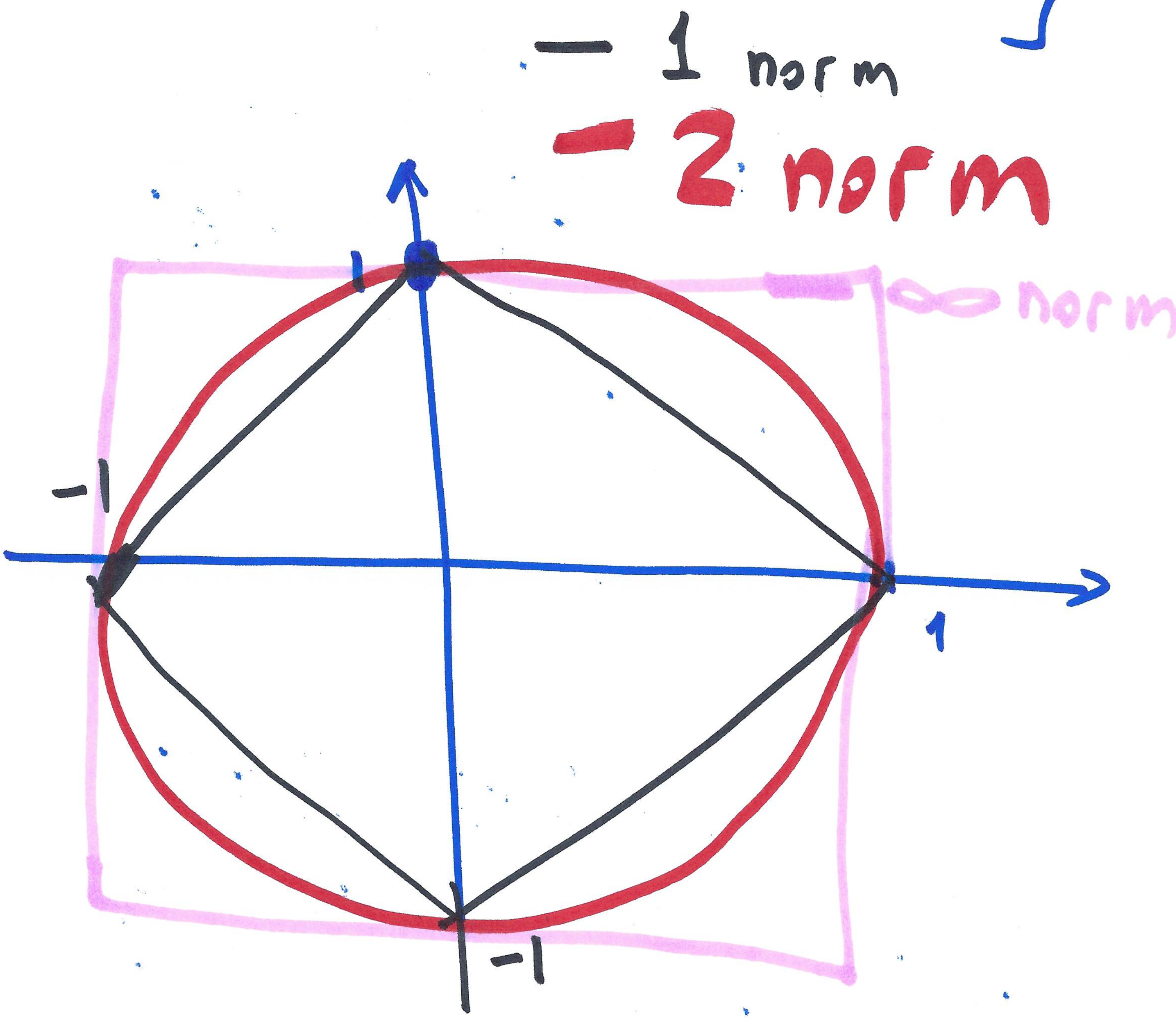
$\underline{x} = (x, y) \qquad \|\underline{x}\| = 1$

$\|\underline{x}\|_1 = |x| + |y| = 1$; ya

$\|\underline{x}\|_2 = \sqrt{x^2 + y^2}$

$\|\underline{x}\|_\infty = \max(|x|, |y|)$

$x \geq 0$, $y > 0$; $x + y = 1$, $y = 1 - x$



$$\|x\|_1 \geq \|x\|_2 \geq \|x\|_\infty$$

K

- $\|x\|_1 \overset{?}{\geq} \|x\|_2 \geq \|x\|_\infty$

$$|a|+|b| \overset{?}{\geq} \sqrt{a^2+b^2}$$

$$a^2+b^2+2|ab| \geq a^2+b^2$$

IF $a=0$, $b^2 = b^2$

$$\|x\|_2 \geq \|x\|_\infty.$$

$$\sqrt{a^2+b^2} \geq \max(|a|,|b|)$$

$$a^2+b^2 \geq \max(|a|^2,|b|^2)$$

$$\|x\|_1 \leq \sqrt{2}\,\|x\|_2 \leq 2\|x\|_\infty$$

$$\|x\|_1 \leq \sqrt{n}\,\|x\|_2 \leq n\|x\|_\infty$$

L

$$(|x| - |y|)^2 \geq \emptyset$$

$$x^2 + y^2 - 2|x| \cdot |y| \geq \emptyset$$

$$x^2 + y^2 \geq 2|x| \cdot |y| \quad \Big| + x^2 + y^2$$

$$2x^2 + 2y^2 \geq x^2 + y^2 + 2|x| \cdot |y|$$

$$\sqrt{2}\sqrt{x^2+y^2} \geq \sqrt{x^2+y^2+2|x||y|} =$$

$$= \sqrt{(|x|+|y|)^2} = |x| + |y|$$

① $\underline{x} \neq \emptyset \Rightarrow \|x\| > \emptyset$

② $\underline{x} = \emptyset \Rightarrow \|x\| = \emptyset$

③ $\underline{x}$ - vector
γ - scalar $\quad \|\underline{x} \cdot \gamma\| = |\gamma| \cdot \|x\|$

④ $\|\underline{x} + \underline{y}\| \leq \|x\| + \|y\|$

M

Matrix norm

$\underline{A}$ - matrix

$$\|A\| = \max_{\forall x \neq \emptyset} \frac{\|A \cdot x\|}{\|x\|}$$

$$\|A\| = \max \|A \cdot x\|, \ \|x\| = 1$$

∞ norm

$$\|x\|_\infty = 1$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\cancel{x = (h, \pm 1), |h| < 1}$

$\cancel{\|A\|_\infty = \max \|A \cdot x\|}$

$x = (h, \pm 1), |h| \le 1$

$x = (\pm 1, h) \ |h| \le 1$

$x = (\pm 1, \pm 1)$

N

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}; \quad \underline{x} = (\pm 1, h)$$

$$\underline{x} = (h, \pm 1)$$

$$\underline{x} = (\pm 1, \pm 1)$$

$$\underline{x} = (x, y)$$

$$\|Ax\|_\infty = \left\| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\|_\infty$$

$$= \left\| \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix} \right\|_\infty$$

$$= \max(ax + by, cx + dy)$$

where $|x| \le 1$

$|y| \le 1$

$$\cancel{= \max(|a| + |d|}$$

$$= \max(|a| + |b|, |c| + |d|)$$

$$\|A\|_\infty = \max_{k=1..n} \sum_{j=1}^{n} |a_{k,j}|$$

Possible to show

$$\|A\|_1 = \max_{k=1..n} \sum_{j=1}^{n} |a_{j,k}|$$

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 9 & 4 \\ 5 & 5 & -5 \end{pmatrix}$$

$$\|A\|_\infty = \max(2+3+1,\ 1+9+4,\ 5+5+5)$$
$$= \max(6, 14, 15) = 15$$

$$\|A\|_1 = \max(2+1+5,\ 3+9+5,\ +1+4+5)$$
$$= \max(8, 17, 10) = 17$$

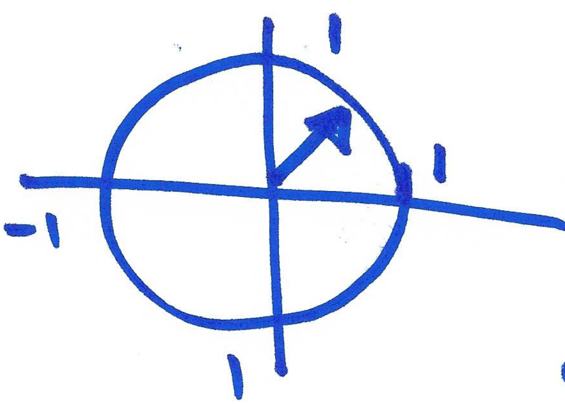
P

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

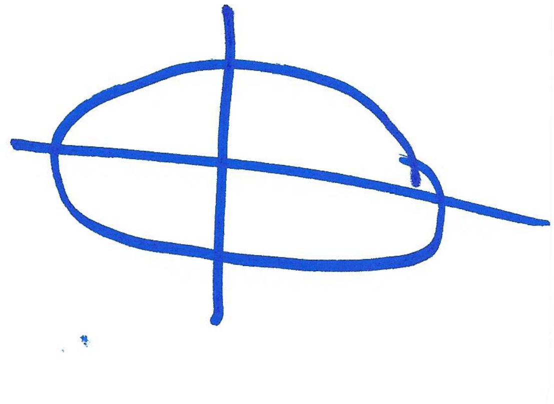
$$\underline{x} = (x, y)\,;\ \|\underline{x}\|_2 = 1 \Rightarrow$$

$$x^2 + y^2 = 1$$

$$y = \pm\sqrt{1 - x^2}$$



$$\Longrightarrow \underline{\underline{A}}\,\underline{x}$$



$$\underline{\underline{A}} \neq 0, \quad \|A\| > 0$$

$$\|\underline{\underline{A}} \cdot \gamma\| = |\gamma| \cdot \|A\|$$

γ is scalar

$$\|\underline{\underline{A}} + \underline{\underline{B}}\| \leq \|A\| + \|B\|$$

$$\|\underline{\underline{A}} \cdot \underline{\underline{B}}\| \leq \|A\| \cdot \|B\|$$

$$\|\underline{\underline{A}}\, \underline{x}\| \leq \|A\| \cdot \|\underline{x}\|$$

$$\|\underline{x}\|_p = \left(\sum_{k=1}^{n} |x_k|^p \right)^{1/p},$$

$p = 1, 2, \infty$

$$\|\underline{\underline{A}}\|_p = \max_{\forall x \neq 0} \frac{\|\underline{\underline{A}}\, \underline{x}\|}{\|x\|}$$

R

26

~~9~~10

26

1

19 70 ~~27~~ 78

92, 31, 30, 22

```
>> LUmine
LUmine
Warning: Function inv has the same name as a MATLAB built-in. We suggest you
rename the function to avoid a potential name conflict.
Compare buid in and the one I wrote based on direct implementation
matrix A
    0.8147    0.0975    0.1576    0.1419    0.6557
    0.9058    0.2785    0.9706    0.4218    0.0357
    0.1270    0.5469    0.9572    0.9157    0.8491
    0.9134    0.9575    0.4854    0.7922    0.9340
    0.6324    0.9649    0.8003    0.9595    0.6787

matlabs LU
Elapsed time is 0.001648 seconds.
Elapsed time is 0.002591 seconds.
resulting A
    0.8147    0.0975    0.1576    0.1419    0.6557
    0.9058    0.1701    0.7954    0.2640   -0.6933
    0.1270    0.5317   -1.5541    0.0682    2.9146
    0.9134    0.8482   -3.6582   -0.8441   -3.2039
    0.6324    0.8892   -3.4808   -0.6838   -0.1376

resulting m
    1.0000         0         0         0         0
    1.1118    1.0000         0         0         0
    0.1559    3.1265    1.0000         0         0
    1.1211    4.9875    2.3539    1.0000         0
    0.7762    5.2288    2.2398    0.8101    1.0000

 Check Matlab accuracy
   1.1102e-16

 Check Heath accuracy
   5.7689e-16

>> x = rand(1000,1);
x = rand(1000,1);
>> norm(x,2)
norm(x,2)

ans =

   17.7143

>> norm(x,3)
norm(x,3)

ans =

    6.1242

>> norm(x,6)
norm(x,6)

ans =

    2.2330

>> norm(x,inf)
norm(x,inf)

ans =
```

```
    0.9995

>> norm(x,inf)
norm(x,inf)
Warning: Function inv has the same name as a MATLAB built-in. We suggest you
rename the function to avoid a potential name conflict.

ans =

    0.9995

>>

>>

>>

>> theta=linspace(0,2*pi,50);
theta=linspace(0,2*pi,50);
>>

>>  theta=linspace(0,2*pi,5)
 theta=linspace(0,2*pi,5)

theta =

         0    1.5708    3.1416    4.7124    6.2832

>> x = [sin(theta),cos(theta)]
x = [sin(theta),cos(theta)]
>>  x = [sin(theta);cos(theta)]
 x = [sin(theta);cos(theta)]
>> theta=linspace(0,2*pi,50);x = [sin(theta);cos(theta)]

theta=linspace(0,2*pi,50);x = [sin(theta);cos(theta)]

>> x = [cos(theta);sin(theta)];
x = [cos(theta);sin(theta)];
>> plot(x(1,:),x(2,:))
```