A

Solve $\underline{\underline{A}}\,\underline{x} = \underline{b}$

Triangular matrix

- → upper
- → lower

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 15 \end{pmatrix}$$

Forward substitute

→ lower triangular matrix

$x_1 = 1$

$2x_1 + 3x_2 = 5 \Rightarrow x_2 = 1$

$4x_1 + 5x_2 + 6x_3 = 15$

$\cancel{6x_3 = 15 - 4 - 5}$

$6x_3 = 15 - 5 - 4 = 6$

$x_3 = 1$

B

C

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{ij} = \begin{cases} \cancel{0}, & j > i \\ \text{nonzero} & \end{cases}$$

lower triangular matrix

a_1

$$\begin{pmatrix} a_{11} & \emptyset & \emptyset \\ a_{21} & a_{22} & \emptyset \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$a_{11} x_1 = b_1 \Rightarrow \boxed{x_1 = b_1 / a_{11}}$$

$$a_{22} x_2 = b_2 - a_{21} x_1$$

$$\boxed{x_2 = \frac{1}{a_{22}} \left(b_2 - a_{21} x_1 \right)}$$

$$a_{33} x_3 = b_3 - a_{31} x_1 - a_{32} x_2$$

$$\boxed{x_3 = \frac{1}{a_{33}} \left(b_3 - a_{31} x_1 - a_{32} x_2 \right)}$$

$k = 1..n$

$$x_k = \frac{1}{a_{kk}}\left(b_k - \sum_{i=1}^{k-1} a_{ki}\, x_i\right)$$

Forward substitution

Backward substitution

- upper triangular matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 6 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \overset{6}{\cancel{0}} \\ -1 \\ -6 \end{pmatrix}$$

$$6x_3 = -6 \Rightarrow x_3 = -1$$

$$4x_2 + 5x_3 = -1$$

$$\cancel{5x_3 = -1 - 4x_2}$$

$$4x_2 = -1 - 5x_3 = -1 + 5 = 4$$

$$x_2 = 1$$

$$x_1 = 0 - 2x_2 - 3x_3$$

$$= -2 + 3 = 1$$

$$\underline{x} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ \emptyset & a_{22} & a_{23} \\ \emptyset & \emptyset & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$x_3 = \frac{1}{a_{33}} b_3$$

Bacboard substitution on upper triangular matrix

$$x_2 = \frac{1}{a_{22}} \left(b_2 - a_{23} x_3 \right)$$

$$x_1 = \frac{1}{a_{11}} \left(b_1 - a_{12} x_2 - a_{13} x_3 \right)$$

$$k = [n : 1]$$

$$x_k = \frac{1}{a_{kk}} \left(b_k - \sum_{i=k+1}^{n} a_{ki} x_i \right)$$

I

A is upper triangular

if $A = (a_{ij})$

$$a_{ij} = \begin{cases} \emptyset, & j < i \\ \text{nonzero} & \text{otherwise} \end{cases}$$

Gauss elimination matrix

$$\begin{pmatrix} \bullet & \bullet \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -\frac{x_2}{x_1} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$$

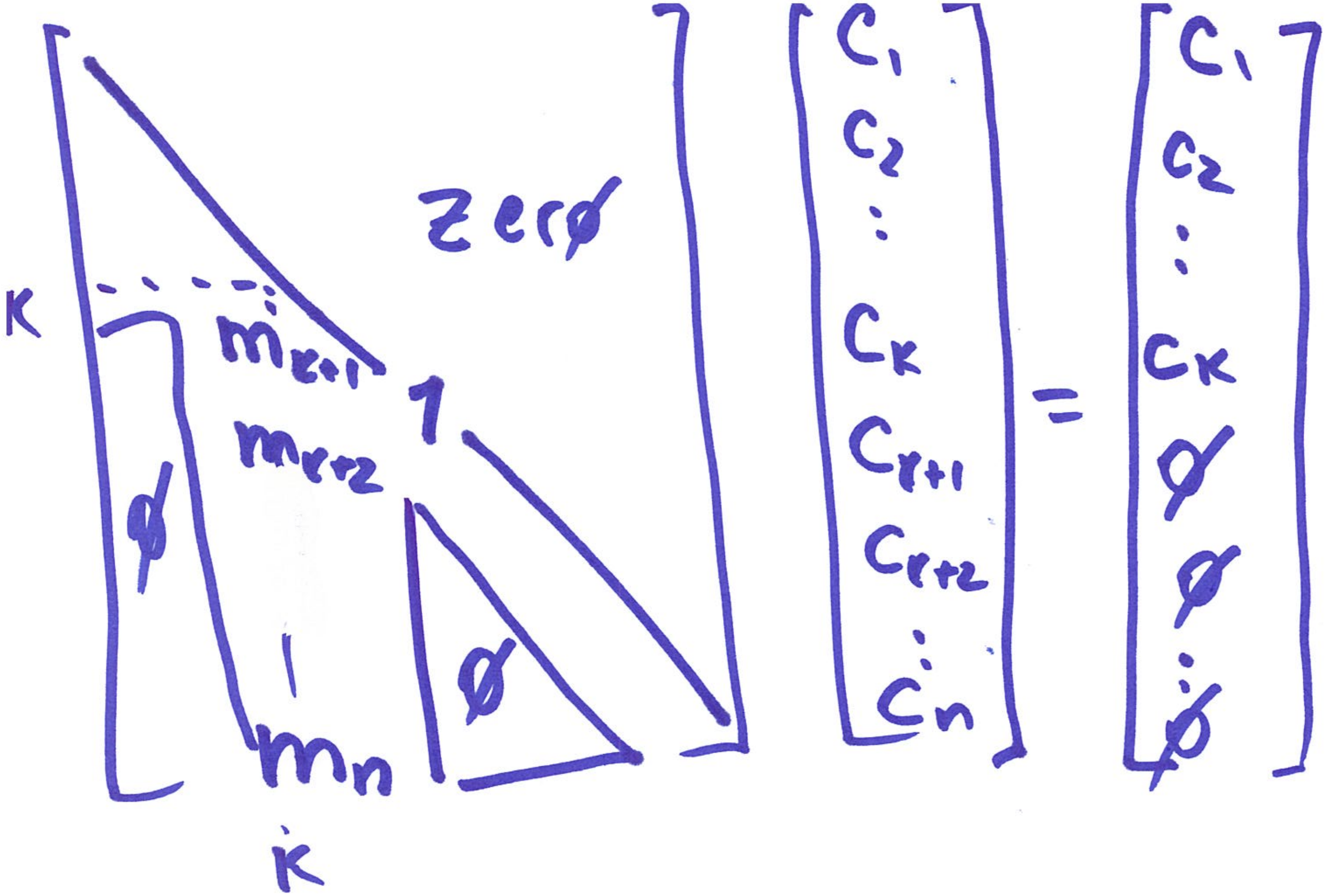
K

$$\begin{pmatrix} \ddots & & \text{zero} \\ m_2 & 1 & \\ m_3 & & \ddots \\ m_4 & \text{zero} & \\ \vdots & & \\ m_n & & \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} c_1 \\ \emptyset \\ \emptyset \\ \emptyset \\ \vdots \\ \emptyset \end{pmatrix}$$

$$m_k = -\frac{c_k}{c_1}$$

$$k = 2..n$$

L



$$m_e = -c_e / c_k$$

$$e = [k+1..n]$$

M

$$\begin{pmatrix} 1 & 0 & 0 \\ -4/5 & 1 & 0 \\ -8/5 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -8/6 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix}$$

N

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -3/2 & 1 & 0 & 0 \\ -5/2 & 0 & 1 & 0 \\ -7/2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7/5 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 5 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -5/3 & 1 & 0 \\ 0 & -7/3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 0 \end{pmatrix}$$

LU decomposition P

L → upper triangular

L → lower triangular

$$(\underline{\underline{A}} \cdot \underline{\underline{B}})^{-1} = \underline{\underline{B}}^{-1} \cdot \underline{\underline{A}}^{-1}$$

$$(\underline{\underline{A}} \cdot \underline{\underline{B}} \cdot \underline{\underline{C}})^{-1} = \underline{\underline{C}}^{-1}\, \underline{\underline{B}}^{-1}\, \underline{\underline{A}}^{-1}$$

$$(\underline{\underline{A}} \cdot \underline{\underline{B}})^{T} = B^{T} \cdot A^{T}$$

P

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & & a_{nn} \end{pmatrix}$$

M_1 eliminates all elements below a_{11}

$$M_1 = \begin{pmatrix} 1 & 0 & & \\ -a_{21}/a_{11} & 1 & & \\ -a_{31}/a_{11} & & 1 & \\ \vdots & 0 & & \ddots \\ -a_{n1}/a_{11} & & & & 1 \end{pmatrix}$$

$$M_1 \cdot A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a'_{22} & & a'_{2n} \\ 0 & a'_{32} & & \vdots \\ \vdots & & & \\ 0 & a'_{n2} & & a'_{nn} \end{pmatrix}$$

$M_1 \cdot A$

Construct M_2 to eliminate all elements below ?, 2

in $M_1 A$

$M_2 \cdot M_1 \cdot A$ will have $\emptyset$ below $(1,1), (2,2)$

Construct M_3 to eliminate all elements below $(3,3)$

..... continue

M_{n-1} which eliminates element below $(n-1, n-1)$

R

$$M_{n-1} M_3 M_2 M_1 A = 21$$

$$M = M_{n-1} M_{n-2} \ldots M_1 =$$

$$= \prod_{k=n-1}^{1} M_k$$

$$(M)^{-1} = (M_{n-1} M_{n-2} \ldots M_2 M_1)^{-1}$$

$$=$$

19,

70

27

78

79