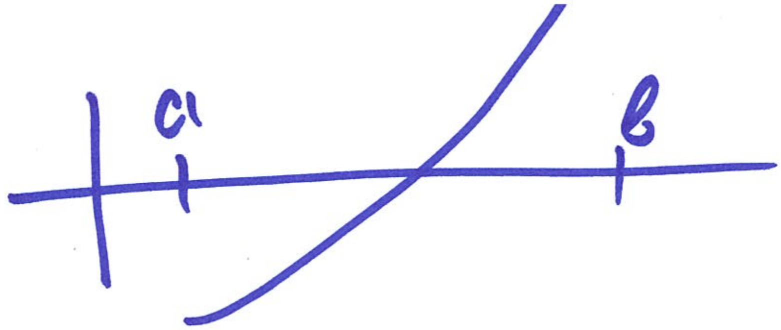
A

evovy@rpi.edu

rensselaer.webex.com
/meet/evovy

Bisection
Newton



B

while $f(a)f(b) < \emptyset$

$m = \dfrac{a+b}{2}$

IF $f(m) = \emptyset$ RETURN m;

IF $f(a)f(m) < \emptyset$, $m = b$

else $m = a$

endif

end while

C

$[b, a]$

$$L_1 = \frac{|b-a|}{2}$$

$$L_2 = \frac{|b-a|}{2^2}$$

$$L_k = \frac{|b-a|}{2^k} = \text{Tolerance}$$

$$\frac{|b-a|}{\text{Tolerance}} = 2^k$$

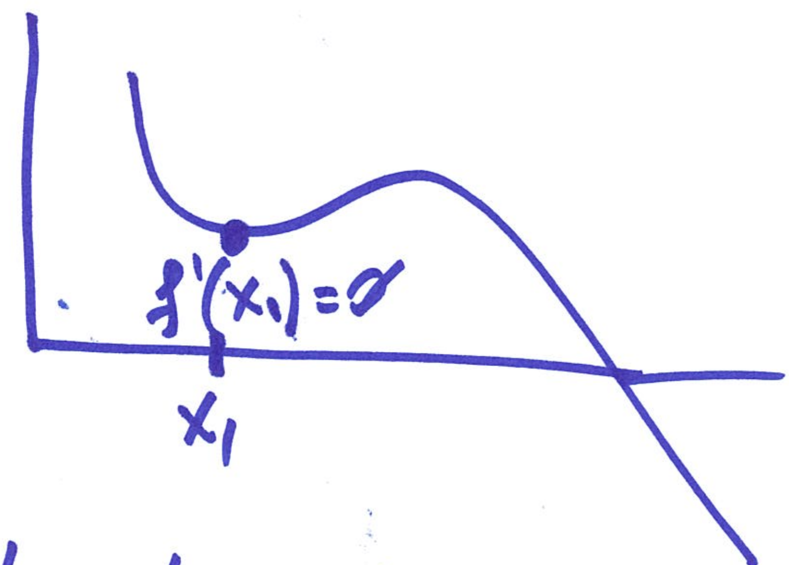
$$\ln\left|\frac{b-a}{\text{Tolerance}}\right| = k \ln 2$$

$$k = \ln\left|\frac{b-a}{\text{Tol}}\right| / \ln 2$$

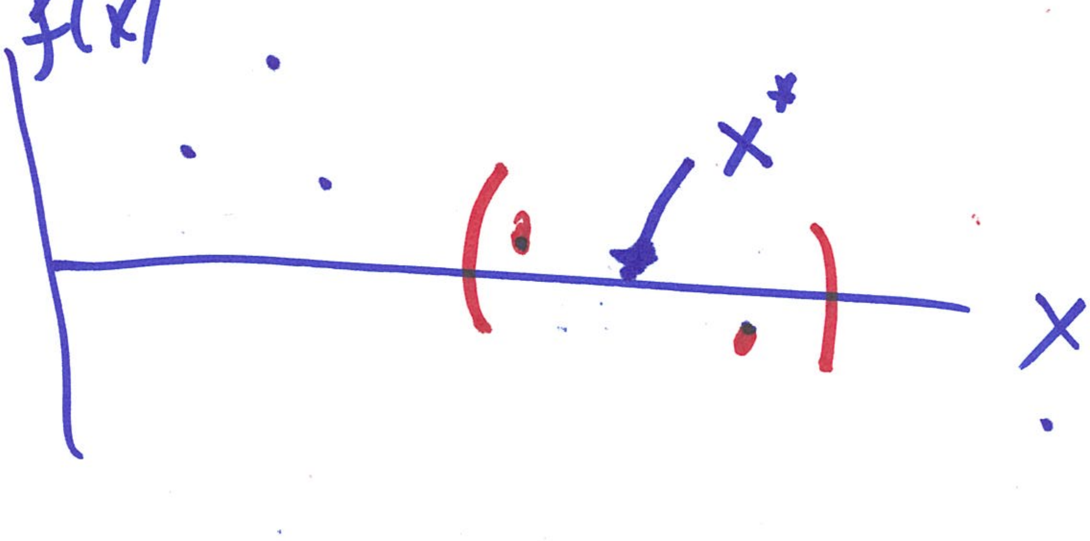
D

Newton's method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \leftarrow \text{Inf} \quad \text{Nan}$$



Start close to
the correct solution



E

Secant method

$$f'(x) = \lim_{\Delta \to 0} \frac{f(x+\Delta) - f(x)}{\Delta}$$

in practice:

$$\Delta = x_k - x_{k-1}$$

$$f'(x) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

How good? $|\Delta| \ll 1$

$$\frac{f(x+\Delta) - f(x)}{\Delta} \cong \frac{f(x) + f'(x)\Delta + \frac{f''(x)\Delta^2}{2} - f(x)}{\Delta}$$

$$= f'(x) + \boxed{f''(x)\Delta/2}$$ Truncation error

F

Newton $$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

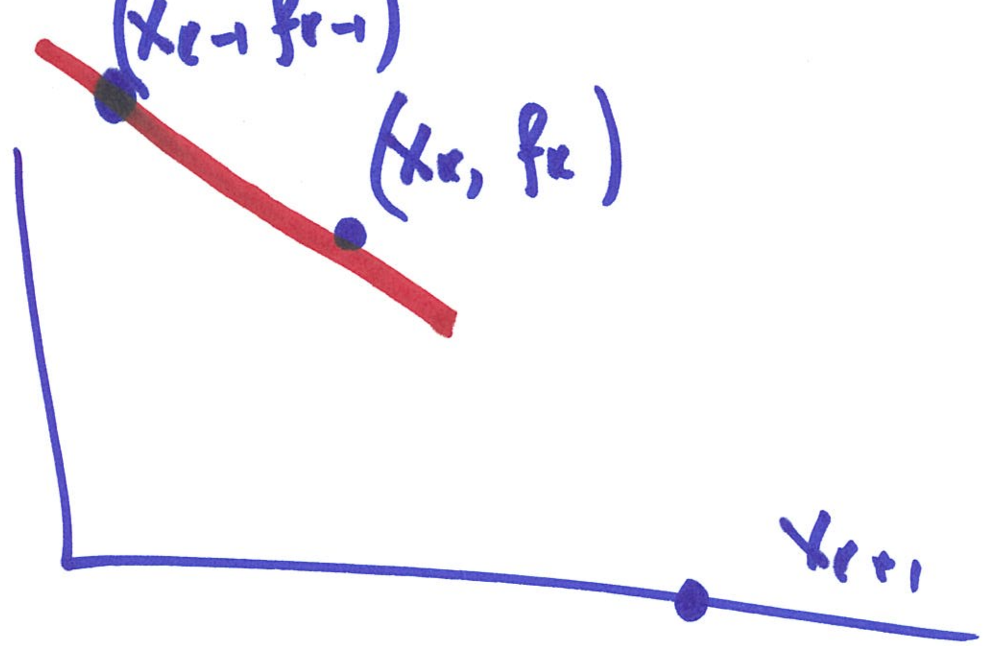
$$f'(x_k) \approx \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

Secant:

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)\,(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}}$$

$$x_{k+1} = \frac{x_k \cdot f(x_k) - x_k f(x_{k-1})}{f(x_k) - f(x_{k-1})} - \frac{f(x_k)x_k - f(x_k)x_{k-1}}{f(x_k) - f(x_{k-1})}$$

$$= \boxed{\frac{f(x_k)\,x_{k-1} - f(x_{k-1})\,x_k}{f(x_k) - f(x_{k-1})} = x_{k+1}}$$



$$f_k \equiv f(x_k)$$

$$f_{k-1} \equiv f(x_{k-1})$$

$$0 = y = ax + b \Rightarrow \boxed{x = -b/a}$$

$$f_{k-1} = a x_{k-1} + b \Rightarrow b = f_{k-1} - a x_{k-1}$$

$$f_k = a x_k + b \qquad b = f_k - a x_k$$

$$f_{k-1} - f_k = a(x_{k-1} - x_k)$$

$$a = \frac{f_k - f_{k-1}}{x_k - x_{k-1}}; \quad b = f_k - a x_k =$$

$$= f_k - \frac{f_k - f_{k-1}}{x_k - x_{k-1}} \cdot x_k$$

$$f_{k-1} - a x_{k-1} = f_k - a x_k$$ H

$$a(x_k - x_{k-1}) = f_k - f_{k-1}$$

$$a = \frac{f_k - f_{k-1}}{x_k - x_{k-1}}$$

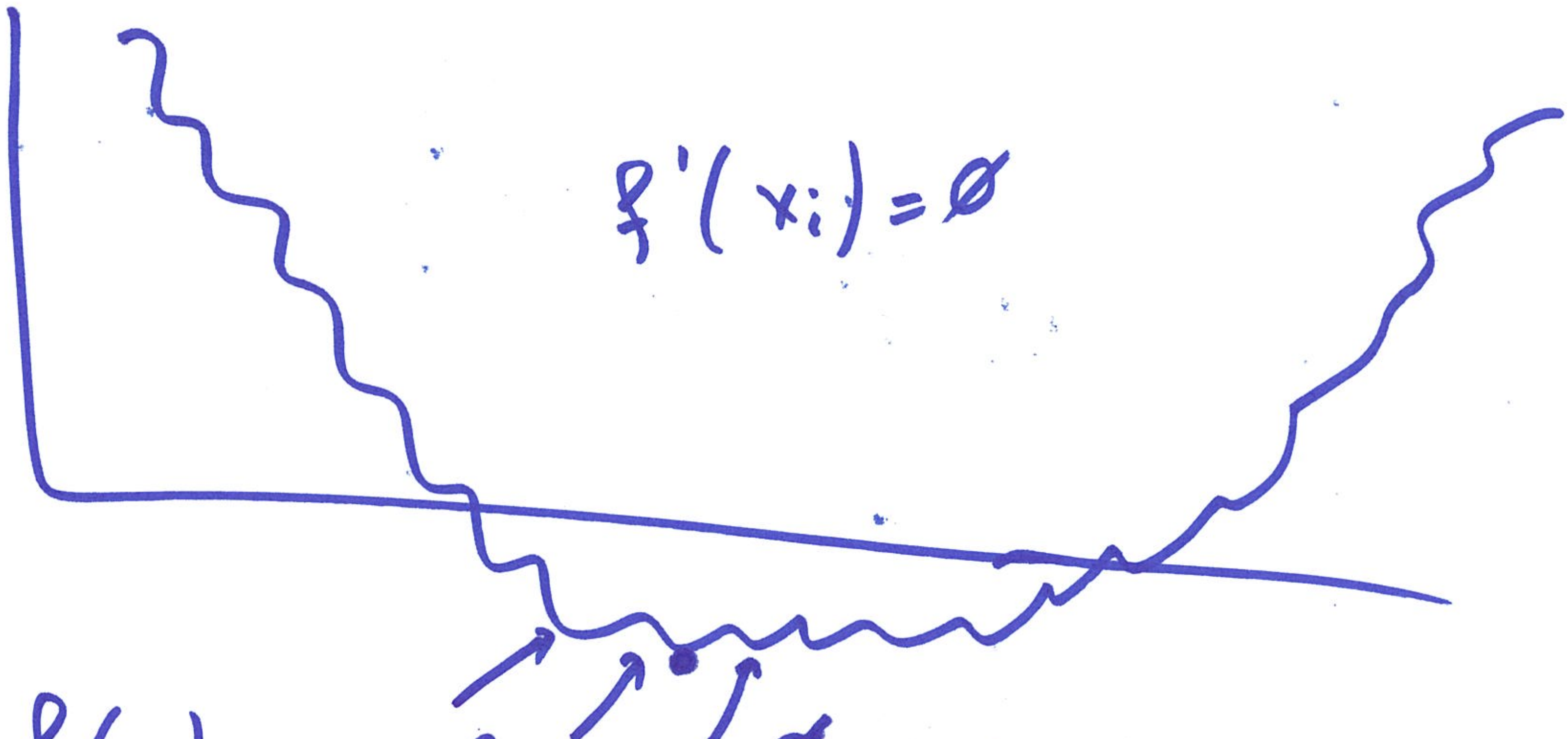
$$b = \frac{f_k x_k - f_k x_{k-1} - x_k f_k + x_k f_{k-1}}{x_k - x_{k-1}}$$

$$= \frac{x_k f_{k-1} - f_k x_{k-1}}{x_k - x_{k-1}}$$

$$x_{k+1} = -\frac{b}{a} = \frac{(-x_k f_{k-1} + f_k x_{k-1})\cancel{(x_k - x_{k-1})}}{\cancel{(x_k - x_{k-1})}(f_k - f_{k-1})}$$

	The Good	The Bad	The ugly	R
Bisection	"Always" works	<u>Very</u> Slowly	Primitive Does not use $f(x)$	
Newton	quadratic convergence	need $f'(x)$ analytically	miss solution altogether must be close to solution	
Secant	superlinear	need 2 starting points	– // –	

5



$$f(x) = (x-5)(x+1) + 0.01\sin(x)$$

T

x_1

x_2

x_3

$\vdots$

x_n

x_{n+1} If $x_n = x_{n+1}$ Then $x_n = x^*$

$$\epsilon_k = |x_k - x^*|$$

error

$$\epsilon_k = C\,\epsilon_{k-1}^{r} \qquad \ln \epsilon_k = \ln\left(C\,\epsilon_{k-1}^{r}\right) =$$

$$\epsilon_{k+1} = C\,\epsilon_k^{r} \qquad = \ln C + r \ln \epsilon_{k-1}$$

$$\ln \epsilon_{k+1} = \ln C + r \ln \epsilon_k$$

$$f(x_{k+1}) = f(x_k + (x_{k+1} - x_k)) =$$

$$= f(x_k) + f'(x_k)(x_{k+1} - x_k) + f''(x_k)(x_{k+1} - x_k)^2 = \emptyset$$

Linear equations

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \dots + a_{1n} x_n = b_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \dots + a_{2n} x_n = b_2$$

$$\cdot \quad - \quad - \quad - \quad - \quad \cdot$$

$$a_{n1} x_1 + a_{n2} x_2 + \dots \qquad + a_{nn} x_n = b_n$$

$$\underline{\underline{A}} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

$$\underline{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} ; \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Solve $\underline{\underline{A}}\,\underline{x} = \underline{b}$; $\underline{x} = \underline{\underline{A}}^{-1} \cdot b$

Find $\underline{x}$

V

$Ax = b$ has unique solution only if

A^{-1} exists

Determinant (A)

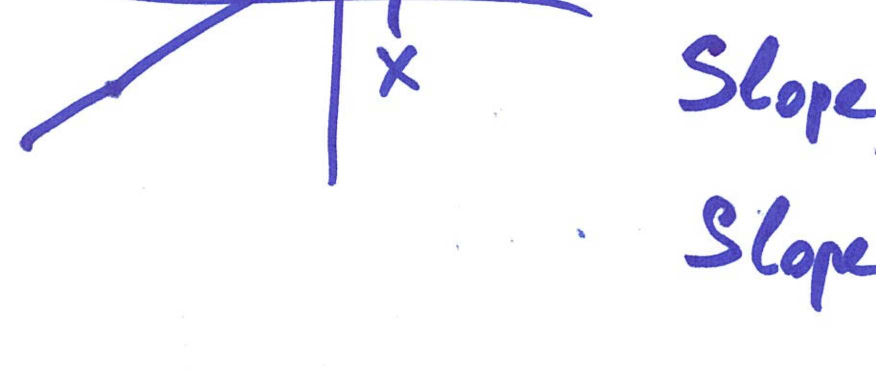
$ax + by = e$

$cx + dy = f$

a, b, c, d, e, f are given

$y = \frac{1}{b}(e - ax)$

$y = \frac{1}{d}(f - cx)$



Slope 1 : $-a/b$

Slope 2 $-c/d$

W

Slope 1 = Slope 2

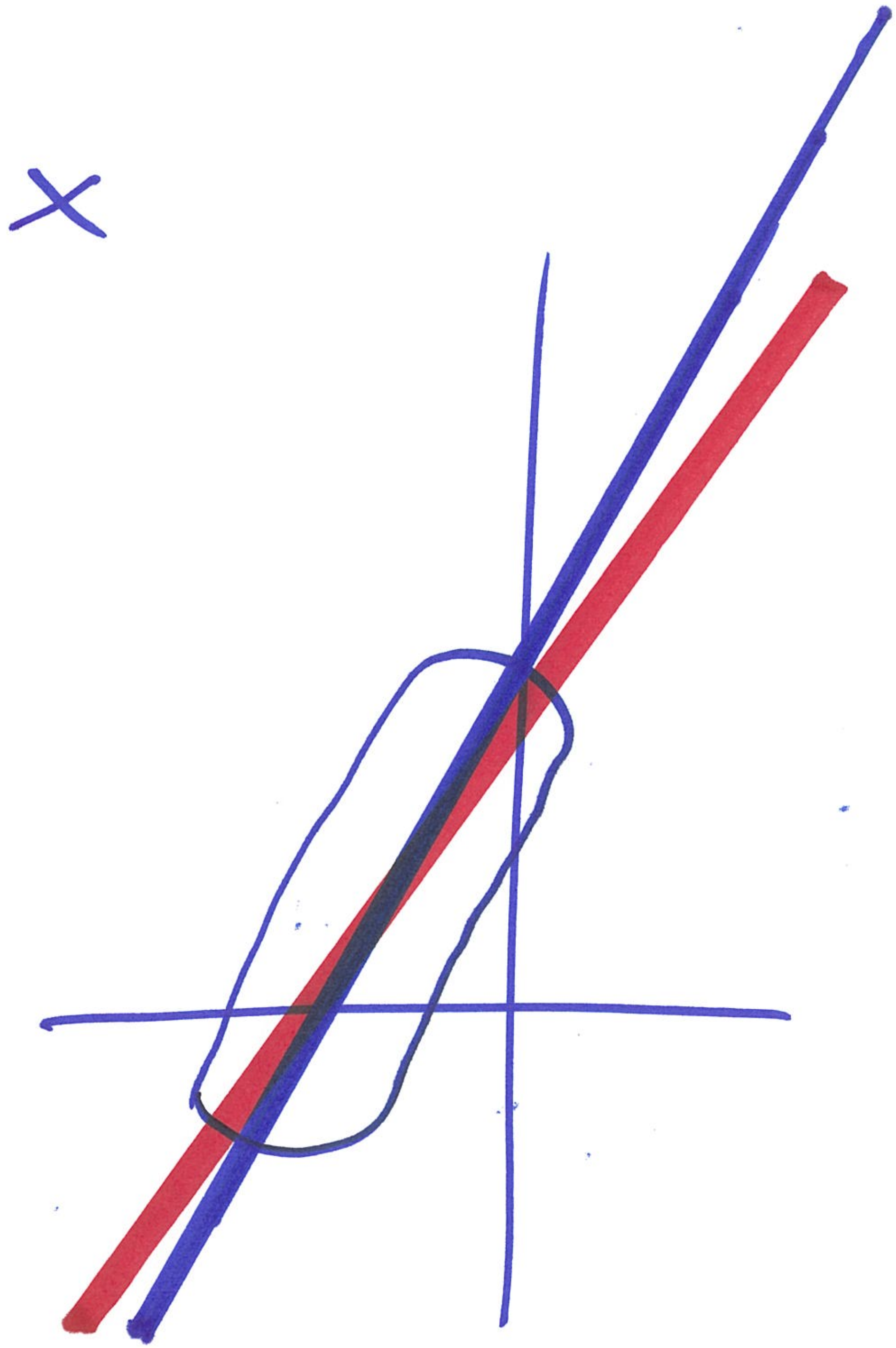
$a/b = c/d$

$\frac{a}{b} = \frac{c}{d}$ $\boxed{ad - bc = \varnothing}$ No solution

$\text{Det} \equiv ad - bc$

Solution exists and is unique if

$\text{Det}(A) \neq \varnothing$



```
Factorial[n] =
{ IF n=1 Answer = 1
  else
    Answer = n. Factorial(n-1)
  end
 return Answer
}
```