A

$\boxed{2}$ (x_1, y_1) $(x_2\ y_2)$

$$f(x) = a_1\, l_1(x) + a_2\, l_2(x)$$

$$l_1(x) = \frac{x_2 - x}{x_2 - x_1}$$

$$l_2(x) = \frac{x_1 - x}{x_1 - x_2}$$

$$l_1(x_1) = 1 = l_2(x_2)$$
$$l_1(x_2) = l_2(x_1) = \emptyset$$

$\boxed{3}$ $x_1\, y_1$ $x_2\, y_2$ $x_3\, y_3$

$l_1(x), l_2(x), l_3(x)$ – Basis functions

$$1 = l_1(x_1) = l_2(x_2) = l_3(x_3)$$
$$l_1(x_2) = l_1(x_3) = l_2(x_1) = \dots$$
$$\dots = l_3(x_2) = \emptyset$$

$$l_i(x_j) = \delta_j^i = \begin{cases} 1, & i = j \\ \emptyset, & i \neq j \end{cases}$$

↳ Kronecker delta

B Lagrange interpolation

2 points $\ell_1(x) = \dfrac{x_2 - x}{x_2 - x_1} = \dfrac{x - x_2}{x_1 - x_2}$

$\ell_2(x) = \dfrac{x_1 - x}{x_1 - x_2} = \dfrac{x - x_1}{x_2 - x_1}$

$$\ell_1(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}$$

$$\ell_2(x) = \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}$$

$$\ell_3(x) = \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}$$

these are polynomials of degree 2

$$\ell_i(x_j) = \delta^i_j = \begin{cases} 1, & i = j \\ \emptyset, & i \neq j \end{cases}$$

$(\emptyset, 1)$ C

$(1,2)$ $y(x) = 1 + x^2$; monomial

$(2,5)$ $x_1 = 0$

$x_2 = 1$

$x_3 = 2$

$$l_1(x) = \frac{(x-1)(x-2)}{(-1)\ (-2)} = \frac{(x-1)(x-2)}{2}$$

$$l_2(x) = \frac{x\ (x-2)}{1\ (1-2)} = x(2-x)$$

$$l_3(x) = \frac{x\ (x-1)}{(2-\emptyset)(2-1)} = \frac{x(x-1)}{2}$$

$$y(x) = \frac{(x-1)(x-2)}{2} + 2\, x(2-x) + \frac{5}{2}(x)(x-1)$$

$$= \frac{x^2}{2} - \underline{\frac{3}{2}x} + 1 + \underline{4x} - 2x^2 + \frac{5}{2}x^2 - \underline{\frac{5}{2}x}$$

$$= x^2 + \emptyset \cdot x + 1$$

$(-1, 0)$ $(0, 1)$ $(1, 4)$ $(2, 15)$ $x_1 = -1$ D

$x_2 = 0$ $x_3 = 1$

$x_4 = 2$

$$e_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{x(x-1)(x-2)}{(-1)(-2)(-3)}$$

$$e_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} = \frac{(x+1)(x-1)(x-2)}{1\cdot(-1)(-2)}$$

$$e_3(x) = \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} = \frac{(x+1)\,x\,(x-2)}{2\cdot 1\,(-1)}$$

$$e_4(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} = \frac{(x+1)\,x\,(x-1)}{3\cdot 2\cdot 1}$$

$$f(x) = \cancel{0}\cdot e_1(x) + e_2(x) + e_3(x) + 15\,e_4(x)$$ $$\boxed{f(x) = 1 + x + x^2 + x^3}$$

E

n points $i=1..n$; (x_i, y_i)

$$\sum_{k=1}^{4} f(x_k) = f(x_1) + f(x_2) + f(x_3) + f(x_4);$$

$$\prod_{k=1}^{4} f(x_k) = f(x_1) \cdot f(x_2) \cdot f(x_3) \cdot f(x_n)$$

$$l_i(x) = \frac{\prod\limits_{\substack{k=1 \\ k \neq i}}^{n} (x - x_k)}{\prod\limits_{\substack{k=1 \\ k \neq i}}^{n} (x_i - x_k)}$$

$i=1..n$

$$f(x) = \sum_{j=1}^{n} y_j \cdot l_j(x)$$

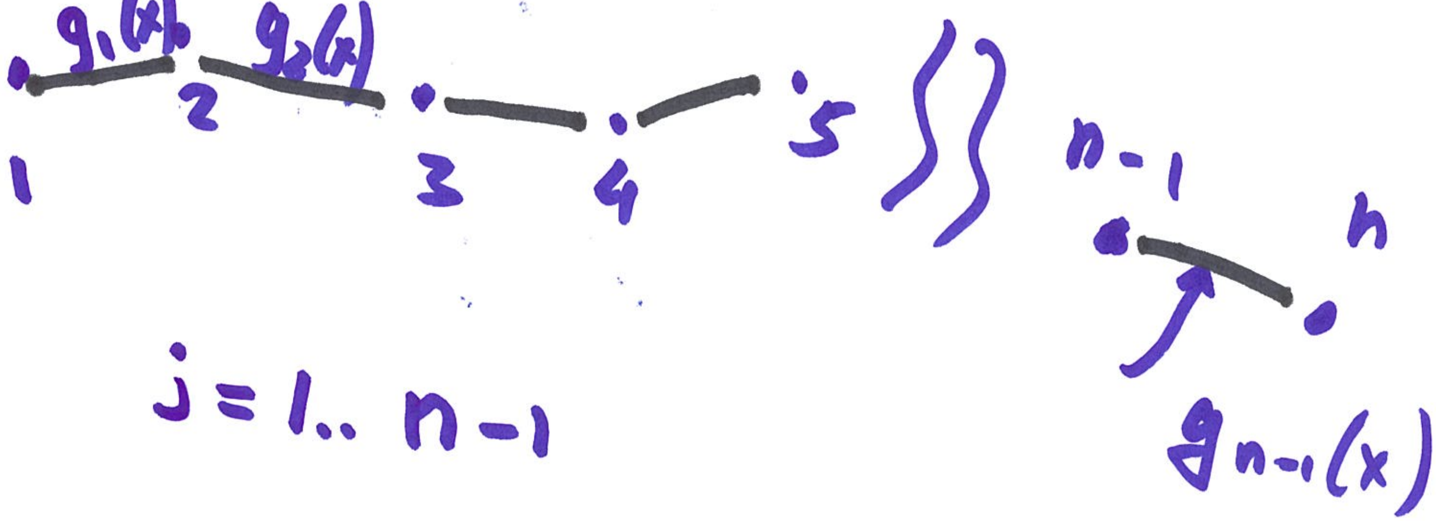
$$l_i(x_j) = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}$$

PW linear interpolation F

n points

$(x_1, y_1)\ (x_2, y_2) \ldots (x_3, y_3) \ldots\ (x_n, y_n)$

$$g_1(x) = y_1 \frac{x_2 - x}{x_2 - x_1} + y_2 \frac{x_1 - x}{x_1 - x_2}$$



$j = 1 \ldots n-1$

$$g_j(x) = y_j \frac{x_{j+1} - x}{x_{j+1} - x_j} + y_{j+1} \frac{x_j - x}{x_j - x_{j+1}}$$

$\bullet\; g_1(x) \;\bullet\; g_2(x)$ $\qquad x_1 < x_2 < x_3 .. < x_5$

$x_1\, y_1 \qquad x_2\, y_2 \qquad \bullet$

$x_3\, y_3$

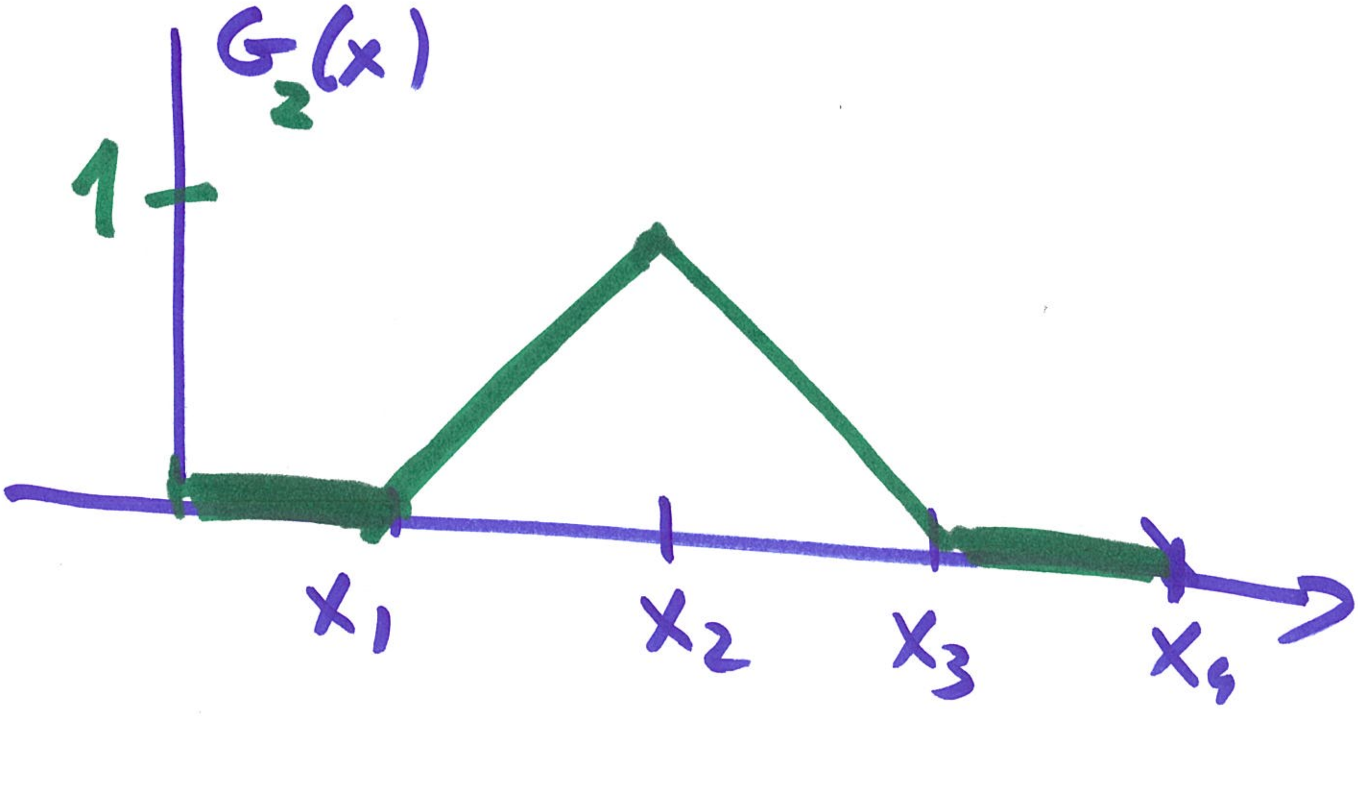
$$g_1(x) = y_1 \frac{x - x_2}{x_1 - x_2} + \boxed{y_2 \frac{x - x_1}{x_2 - x_1}}$$

$$g_2(x) = \boxed{y_2 \frac{x - x_3}{x_2 - x_3}} + y_3 \frac{x - x_2}{x_3 - x_2}$$

$$G_2(x) = \begin{cases} 0, & x \le x_1 \\ \dfrac{x - x_1}{x_2 - x_1} & x_1 \le x \le x_2 \\ \dfrac{x - x_3}{x_2 - x_3} & x_2 \le x \le x_3 \\ 0 & x \ge x_3 \end{cases}$$

H

$$G_2(x) = \begin{cases} 0, & x \le x_1 \\ \dfrac{x - x_1}{x_2 - x_1} & x_1 \le x \le x_2 \\ \dfrac{x - x_3}{x_2 - x_3} & x_2 \le x \le x_3 \\ 0 & x \ge x_3 \end{cases}$$



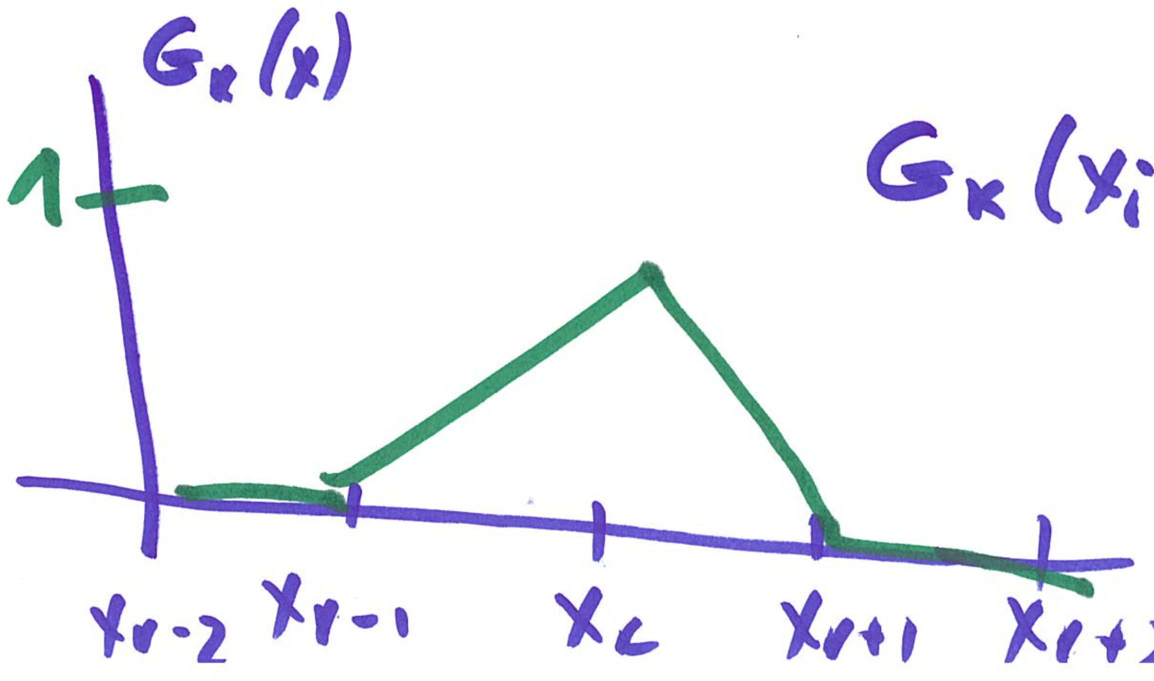
I

$(x_i\ y_i)\ i=1..n$;

n functions $G_k(x)$

$k=1..n$

$$G_k(x)=\begin{cases} \emptyset, & x<x_{k-1}\\[2ex] \dfrac{x-x_{k-1}}{x_k-x_{k-1}} & x_{k-1}\le x\le x_k \\[2ex] \dfrac{x-x_{k+1}}{x_k-x_{k+1}} & x_k\le x\le x_{k+1}\\[2ex] \emptyset & x\ge x_{k+1}\end{cases}$$



$$G_k(x_i)=\begin{cases} 1, & i=j\\ \emptyset & i\neq j\end{cases}$$

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$$f(x) = \sum_{k=1}^{n} y_k G_k(x)$$