A

ypa

$$\underline{f}(\underline{x}) = 0,$$

$$\underline{f}(\underline{x}_{k+1}) = \underline{f}(\underline{x}_k + \underline{x}_{k+1} - \underline{x}_k)$$

$$= \underline{f}(\underline{x}_k) + J(\underline{x}_{k+1} - \underline{x}_k) = 0$$

$$\partial f_i / \partial x_j$$

$$J(\underline{x}_{k+1} - \underline{x}_k) = -\underline{f}(\underline{x}_k)$$

$$\underline{x}_{k+1} = \underline{x}_k - \left(J(\underline{x}_k)\right)^{-1} \underline{f}(\underline{x}_k)$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$\underline{\underline{A}} \in R^{n \times n}$

$\underline{x} \in R^n$ such That

$$\underline{\underline{A}} \cdot \underline{x} = \lambda \cdot \underline{x}$$

λ → scalar

λ → eigenvalue

$\underline{x}$ → eigenvector

$$\underline{\underline{A}}\,\underline{x} = \lambda \underline{x}$$

$$\underline{\underline{A}}\,\underline{x} - \underline{x}\lambda = \emptyset$$

$$\underline{\underline{A}}\,\underline{x} - \lambda \underline{\underline{I}} \cdot \underline{x} = \emptyset$$

$$\left(\underline{\underline{A}} - \lambda \underline{\underline{I}}\right)\underline{x} = \emptyset$$

$$\mathrm{Det}\left(\underline{\underline{A}} - \lambda \underline{\underline{I}}\right) = \emptyset \Rightarrow n \text{ eigenvalues}$$

$$\underline{\underline{A}}\,\underline{x}_i = \lambda_i \underline{x}_i$$

C

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$\lambda_1 = +1, +2, +3;$

$$\mathrm{Det}(\underline{\underline{A}} - \lambda \underline{\underline{I}}) =$$

$$= \mathrm{Det}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] =$$

$$= \mathrm{Det}\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} =$$

$$= (1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$\lambda_1 = 1; \ \lambda_2 = 2; \ \lambda_3 = 3;$

C

$\lambda_1 = 1; \quad \underline{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$\underline{\underline{A}}\,\underline{x} = \lambda_1 \cdot \underline{x}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$a = a \qquad a = 5 \; ; \quad a = 1$

$2b = b \qquad b = 0$

$3c = c \qquad c = 0$

$\underline{x}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \underline{x}_1' = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$

$\lambda_2 = 2; \quad \underline{x}_2 = \begin{pmatrix} d \\ e \\ f \end{pmatrix}; \quad d = f = 0$

$e = 1 \text{ or } 5$

$\underline{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad \lambda_3 = 3; \; x_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

D

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\text{Det}(A - \lambda \underline{I}) =$$

$$= \text{Det}\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} =$$

$$= (2-\lambda)^2 - 1 = 0$$

$$(2-\lambda)^2 = 1$$

$$(\lambda-2)^2 = 1$$

$$\lambda - 2 = \pm 1$$

$$\lambda = 3, 1$$

$\lambda_1 = 3$ E

- $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 3\begin{pmatrix} a \\ b \end{pmatrix}$

$2a + b = 3a \Rightarrow a = b$

$a + 2b = 3b \Rightarrow a = b$

$\underline{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$; $\underline{x} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

- $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$

$2c + d = c$; $d = -c$

$c + 2d = d$; $c = -d$

$\underline{x}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

F

$$A = \begin{pmatrix} 1 & 2 \\ -1/2 & 1 \end{pmatrix}$$

$\mathrm{Det}(A - \lambda I) = (1-\lambda)^2 + 1 = \emptyset$

$(\lambda - 1)^2 = -1$; $\lambda - 1 = \pm i$; $\lambda = 1 \pm i$

If $\mathrm{Im}(A) = \emptyset$,

$A x = \lambda x$

then $A x^* = \lambda^* x^*$

if λ is eigenvalue, x is an eigenvector

then λ^* and x^* are also eigenvalue and eigenvector

G

$$\begin{pmatrix} 1 & 2 \\ -1/2 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = (1+i)\begin{pmatrix} a \\ b \end{pmatrix}$$

$$a + 2b = a + ia$$

$$2b = ia$$

$$-i2b = a; \quad \underline{x} = \begin{pmatrix} -2i \\ 1 \end{pmatrix}$$

$$\underline{x} = \begin{pmatrix} -2ib \\ b \end{pmatrix}$$

$$\|\underline{x}\|_2 = \sqrt{4b^2 + b^2} = \sqrt{5}\, b$$

$$b = \frac{1}{\sqrt{5}} \quad \underline{x} = \begin{pmatrix} -2i/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$ H

$$\operatorname{Det}(A - \lambda \underline{I}) = (2-\lambda)^2 = \emptyset$$

$$\lambda_2 = \lambda_1 = 2;$$

$$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 2\begin{pmatrix} a \\ b \end{pmatrix}$$

$$2a + b = 2a, \quad b = \emptyset$$

$$2b = 2b$$

$$\underline{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

IF A is symmetric
($A^T = A$)

then

- its eigenvalues are real
- IF x_i and x_j are two eigenvectors corresponding to eigenvalues $\lambda_i \neq \lambda_j$, then $\underline{x}_i^T \cdot \underline{x}_j = 0$
- it is possible to find orthonormal set such that

$$x_i^T \cdot x_j = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

IF A is $n \times n$ matrix and

$\lambda_1 \dots \lambda_n$ are its eigenvalues

and $|\lambda_1| > |\lambda_j|$; $j = 2 \dots n$

then λ_1 is "dominant" eigenvalue.

Ex: $A = \begin{pmatrix} 1 & 0 \\ 0 & -9 \end{pmatrix}$; -9 is dominant eigenvalue.

Power method

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad \begin{array}{l} \lambda_2 = 1 \; ; \; x_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ \lambda_1 = 3 \; ; \; x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{array}$$

K

$$x^T \left(A x = \lambda x \right)$$

$$x^T A x = \lambda x^T x$$

$$\boxed{\lambda = \frac{x^T A x}{X^T \cdot x}}$$

$$\underline{y}_0 = (y_{0x}, \; y_{0y})$$

$$y_1 = A \cdot y_0$$

$$y_2 = A y_1 = A^2 y_0$$

$$y_3 = A y_2 = A^3 y_0$$

$$- - - -$$

$$y_k = A^k y_0 \; ; \quad k = 1, 2, 3 \ldots\ldots$$

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}; \quad \underline{x}_1, \lambda_1$$

$$\underline{x}_2, \lambda_2$$

$$y_0 = \alpha_1 x_1 + \alpha_2 x_2;$$

$$y_1 = A y_0 = A(\alpha_1 x_1 + \alpha_2 x_2) =$$

$$= \alpha_1 A x_1 + \alpha_2 A x_2 =$$

$$= \lambda_1 \alpha_1 x_1 + \alpha_2 \lambda_2 x_2;$$

$$y_2 = A y_1 = A(\alpha_1 A x_1 + \alpha_2 A x_2) =$$

$$= \lambda_1^2 \alpha_1 x_1 + \alpha_2 \lambda_2^2 x_2$$

$$y_k = \lambda_1^k \alpha_1 x_1 + \alpha_2 \lambda_2^k x_2; \quad \lambda_2 < \lambda_1$$

$$y_k = \lambda_1^k \left(\alpha_1 x_1 + \alpha_2 \left(\frac{\lambda_2}{\lambda_1}\right)^k x_2\right)$$

$$\lim_{k \to \infty} y_k = \lim_{k \to \infty} A^k y_0 =$$

$$= \lambda_1^k \alpha_1 x_1$$

A is $R^{n \times n}$ M

$\lambda_1 \ldots \lambda_n$ $|\lambda_1| > |\lambda_2| > \ldots > |\lambda_n|$

$x_1 \ldots x_n$

$$\underline{y}_0 = \sum_{k=1}^{n} \alpha_k \underline{x}_k$$

$$\underline{y}_1 = \underline{\underline{A}}\,\underline{y}_0 = A \sum_{k=1}^{n} \alpha_k \underline{x}_k = \sum_{k=1}^{n} \alpha_k \lambda_k \underline{x}_k$$

$$\underline{y}_m = \underline{\underline{A}}\,\underline{y}_{m-1} = \sum_{k=1}^{n} \alpha_k \lambda_k^m \underline{x}_k = \alpha_1 \lambda_1^m \underline{x}_1 + \sum_{k=2}^{n} \alpha_k \lambda_k^m \underline{x}_k = \lambda_1^m \left(\alpha_1 x_1 + \sum_{k=2}^{n} \alpha_k \left(\frac{\lambda_k}{\lambda_1}\right)^m \underline{x}_k \right)$$

$$\lim_{m \to \infty} y_m = \lim_{m \to \infty} A^m y_0 = x_1 \cdot \alpha_1 \lambda_1^m$$

$x_1^T x_2 = x_2^T x_1 = \cancel{0}$

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$\lambda_2 / \lambda_1 = \omega$

$$y_0 = \alpha_1 x_1 + \alpha_2 x_2 ;$$

$$y_k = \lambda_1^k \left(\alpha_1 x_1 + \alpha_2 \underbrace{\left(\frac{\lambda_2}{\lambda_1}\right)^k}_{\omega^k} x_2 \right) =$$

$$= \lambda_1^k \left(\alpha_1 x_1 + \alpha_2 \omega^k x_2 \right)$$

v_k - current approximation to eigenvalue

$$v_k = \frac{y_k^T \cdot A \cdot y_k}{y_k^T \cdot y_k} =$$

$$= \frac{\lambda_1^{2k} (\alpha_1 x_1^T + \alpha_2 \omega^k x_2^T) A (\alpha_1 x_1 + \alpha_2 \omega^k x_2)}{(\alpha_1 x_1^T + \alpha_2 \omega^2 x_2^T)(\alpha_1 x_1 + \alpha_2 \omega^k x_2)}$$