Numerical Computing
MATH 4800 Spring 2024

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Tuesday-Friday 12:00-13:50
Office Hours : Mondays, 18:00-20:00, webex personal room
Feel free to talk with me *after* lectures
Best way to contact me is email lvovy@rpi.edu.[1]

MIDTERMS are February (?)2, March 15 and April 26, 2024
Matlab Laboratory is April 19, 2024
yurilvov.com/NC
TA: Miles Corn *cornm@rpi.edu* Friday 11am-1pm, AE423

1 General Info

Course objective Introduction to numerical methods for scientific and engineering problems. Hands on experience with Numerical methods, Matlab, error analyses.

. If computer gives you an answer, the answer is **always** approximate. After this course you will understand why it is so, and will be able to minimize the error.

1.1 Student Learning Outcomes: Upon successfully completing the courses, students will be able to:

Be proficient in basical numerical methods, being able to solve numerically and analyze errors for linear systems, least squares, nonlinear equations, interpolations, numerical integration and differentiation, numerical solutions of ODE's.

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[1]If you do not hear back from me by the end of the next business day, please resend your email

1.2 Outline

1. Introduction, errors, and FLOPS
   Chapter 1   3 lectures
   • Approximation and Error • Computer arithmetic

2. Nonlinear Equations
   Chapter 2   4 lectures
   • Introduction and scalar equations • Nonlinear system of equations

3. Numerical solution of Linear systems
   Chapter 3   4 lectures
   •Solving linear systems •Error analyses and accuracy •iterative methods

4. Eigenvalue Problem
   Chapter 4

5. Interpolation *Chapter 5*   3 lectures
   • Polinomial interpolation • Piecewise polyunomial interpolation

6. Numerical Integration
   Chapter 6   3 lectures

7. Numerical solution of the ODE's
   Chapter 7   4 lectures
   • Numerical quadrature • Numerical differentiation

   • Introduction and Euler's method • Accuracy and stability • Implicit method and stiff equations
   • Runge-Kutta and Taylor methods

8. Least squares
   Chapter 8   3 lectures

   28 lectures including 3 midterms and one matlab lab

1.3   Books and other prerequisites

1. Book: *Introduction to Scientific Computing and Data Analysis (Texts in Computational Science and Engineering, 13)*
   2nd ed. 2023 Edition

   by Mark Holmes, –REQUIRED

   Text book

   ISBN-10-3031224299

   amazon 83 USD

2. Scientific Computing: An Introductory Survey by Michael Heath. ISBN: 978-1-611975-57-4 Second edition Recommended

3. Willingness to work in Matlab

   Guides to Matlab:

   - For detailed introduction see
     Numerical Computing with Matlab
   - A Practical Introduction to Matlab by Mark S. Gockenbach
   - Matlab Academy
     Need RPI account

4. Wikepedia

Alternative to Matlab Freely available packages named "Octave" (octave.org) and Scilab (scilab.org) offer functionality similar to that of Matlab. You may find these free options to be apealing, especially for off-campus use.

1.4   Grade Policy

Homework projects **25 %** + 2 bestMidterm Exams out of 3 **30 %** + Matlab Laboratory **10%**Final Exam+ **35 %**+

Optional, subject to availability:[2] *Final Project +10 %*
"Cell Phone" usage in class -5 percent of the final grade
I will assign a grade based on the following:
$90 \leq g \leq 100$ is A
$80 \leq g < 90$ is B
$70 \leq g < 80$ is C
$67 \leq g < 70$ is D
$g < 67$ is F

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[2]This option is available ONLY if your grade is above 70 percent

Grades modifiers will be used: $0 \leq g \leq 3$ is "-", $3 < g \leq 6$ has no modifier, $6 < g$ is "+".

There is no "make up" policy for homeworks. **ALL HOMEWORKS ARE REQUIRED** Since two best midterm exams are taken into account, you may miss one exam. If you miss two or three exams, exams are replaced by an oral exam to the instructor (not recommended).

Late Homeworks: 10 % off per each **complete 24 hours** starting at the deadline. *This policy is strictly enforced*

I did hear stories about dogs eating homeworks.

There is no "make up" policy for homeworks or exams.

1.5   Few notes, policies, etc

There is no "curve grading".

Attendance is strongly encouraged and may be **logged** You are responsible for knowing what was said in class. Use class webpage at your own risk.

Usage of **LAPTOPS, CELL PHONES, or lightsabers.** will result in 5 percent substracted from your final grade.

ıIf you are late to class, you agree to either dance, tell a joke or sing.

This course may be the hardest course you have ever taken, since there are about 7 homeworks, each one of them takes about 6-10 hours, depending upon how fast you can code. There are four exams, (three midterms and one final) and one Matlab Laboratory, each of which requires preparation.

My promise: Those with passing grade will be proficient with Numerical Programming. There is a way to the mountain top ready to be shown to you. *You* have to do the walking.

Although some MATLAB basics will be presented, this *is not* the course about MATLAB usage. Therefore you should either be familiar with MATLAB, or be willing to learn it.

Academic Integrity

The grade you receive for the course will be based on the work that you do.

With this principle in mind, the work (exams, homework, computer programs) that you present for a grade MUST, in fact, be of your own.

With respect to the exams, this means that no assistance or collaboration of any kind is permitted (other then assisace obtained from the instructor). Anyone violating this policy will receive an exam grade of zero and will be reported to the Dean of Students.

With respect to problem sets (homeworks and or computer projects), you are free to seek assistance or advise from any person, book or computer. However, what you hand in must be your own work. In this regard, **computer files must not be shared or exchanged** nor should you copy work from someone else. If you do collaborate, please **provide names** of people you worked with at the beginning of your assignment. Violating of this policy will result in a score of zero for the course, and will be reported to the Dean of Students.

Note that your health, need of financial aid, need to maintain GPA, need to graduate, obtain employment, etc will not be considered.

I will not provide tutoring to you.

BUT if you sense you have a problem, please talk to me. I may be able to help!

Neither instructor nor the TA is responsible for briefing you on the context of missed lectures

Important Note on HW's

For the computing portions of the assignments, it is not enough to print out pages of output and code without commentation and little discussion. Instead, to get a full credit, it is **required** to include two important things (1) a 1-2 paragraph summary of your results and (2) your results (including convergence and error results) in tabular format. This will allow you to gather your ideas for better performance.

It is your responsibility to present your Homework in a best possible way. Please write neatly (better type), and respect grader/instructor time. Do not expect grader/instructor to figure out what you meant. Make it crystal clear!

1.6 Optional Final Project

Optional final project is to use skills learned in class to *solve* innovative interesting problem of your own choice. You will need to write *your own* matlab code to solve this problem. You need to make inclass 15 minutes presentation and submit a final written report on the topic of your choice where you use methods of numerical computing to solve some cool problem.

Time line for final project:

- **stage one** - you think about what you want to do, talk to your friends, talk to me, bounce ideas of the wall - February 1, 2024

- **stage 2**- you submit to me a title and a one page description of your project **February 150, 2024**

- **stage 3** I choose 3 or 4 best projects and will notify you if you are selected.

- **stage 4** - you submit to me transparencies of your talk in doc or power-point format **March 15, 2024**.

- **stage 4** you give in-class presentation and answer questions **April 15 2024**

- Last day of classes - you submit your written report in **LATEX** format

Submission of your final report constitutes your agreement to wave all the copyrights of your report. It may be made available to other NC students.
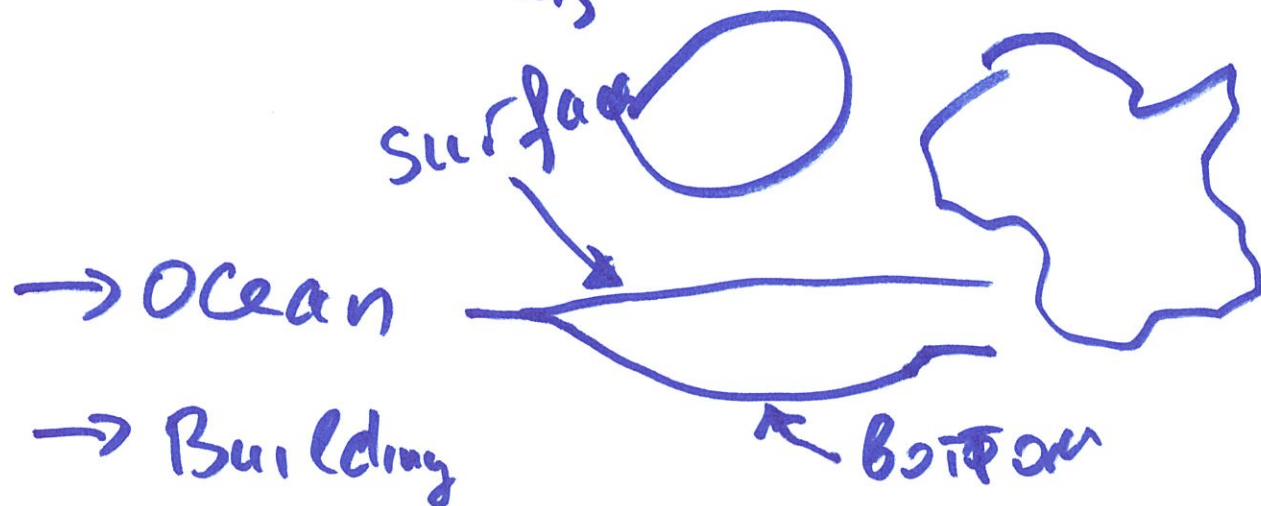
Welcome A

to

Num Comp

Area of Earth

$$A = 4\pi \cdot R^2$$

$$R - radius \qquad \pi$$

$$R = 6400 \; km$$

→ Earth is NOT a sphere

→ Radius is impresize

→ maintains

surface

→ Ocean

→ Building   ↖ Bottom

X - real value

X_M - computer gives you

Absolute Error $= |X - X_M|$

Relative Error $= \left| \dfrac{X - X_M}{X} \right|$

per. cents

E

- $x = 3 \cdot \left(\dfrac{4}{3} - 1\right) - 1 = \cancel{0}$
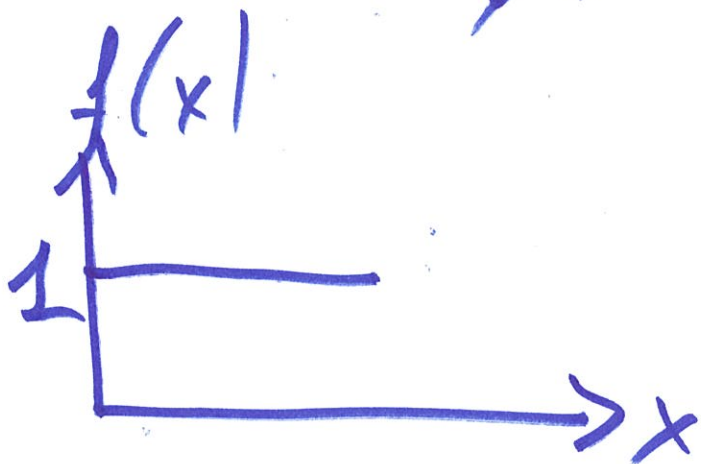
- $S = \displaystyle\sum_{k=1}^{\infty} \dfrac{1}{k}$

Infinity

$$\cancel{\infty} \leftarrow 10^4$$

$S_1 = \displaystyle\sum_{k=1}^{10^4} \dfrac{1}{k}$

$S_2 = \displaystyle\sum_{k=10^4}^{1} \dfrac{1}{k}$

Mathematically $S_1 = S_2$

F

$$f(x) = \frac{1 - (1-x)}{x}$$

Binaries

hundred's place

$$6893 = 6 \cdot 10^3 + 8 \cdot 10^2 + 9 \cdot 10 + 3$$

thousand's place

One's place

ten's place

Base 10

$$3.1415 = $$

$1/100$

$1/10000$

$1/10$

$1/1000$

$$= 3 + \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} + \frac{5}{10000}$$

Base 2

$$1010.01_{[2]} =$$

two, ones, ¼, four, eighth
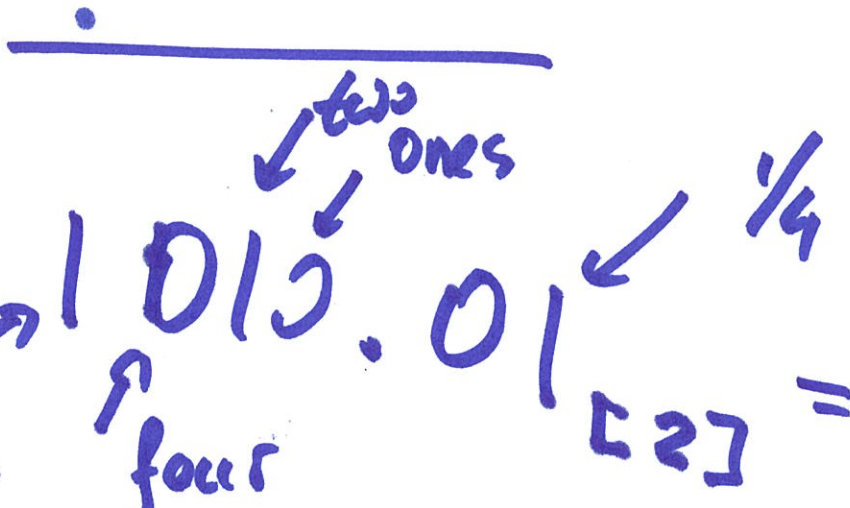
$$= \emptyset \cdot 1 + 1 \cdot 2 + 0 \cdot 4 + 1 \cdot 8$$
$$+ \, {}^0\!/_2 + {}^1\!/_4 \, {}_{[10]} =$$
$$= 2 + 8 + {}^1\!/_4 = 10.25$$

Floating Point System

β - base

N: - precision
number of digits

$$X = \pm \left(d_1 + \frac{d_2}{\beta} + \frac{d_3}{\beta^2} + \dots + \frac{d_N}{\beta^{N-1}} \right) \cdot \beta^E$$
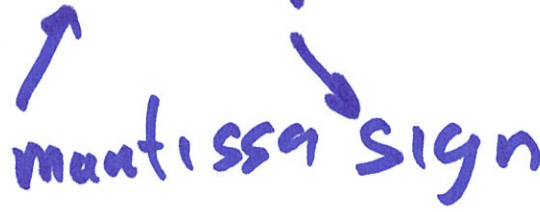
Normalized
$d_1 \neq \emptyset$

$d_1 d_2 \dots d_N$
mantissa

E
-expo

$d_i = \emptyset \dots \beta - 1$ - integer

E is an integer

$L \leq E \leq u$

	β	N #of digits	L	U
"Human"	10	3	-3	3
Single precision	2	24	-126	127
Double precision	2	53	-1022	1023
Toy	2	3	-1	1

Double $53 + 1 + 11 = 65$ bits

mantissa

sign

$d_1 = 1$

normalized

Toy system

$$\beta = 2, \quad N = 3,$$

$$L = -1, \quad U = 1$$

Write all numbers
positive

$$X = \pm \left(d_1 + \frac{d_2}{2} + \frac{d_3}{4} \right) 2^E$$

Normalized $d_1 = 1$

mantissa's

1.00

1.01

1.10

1.11

L

$\beta = 2$	$\beta = 10$
1.00	1
1.01	$1 + \frac{1}{4} = \frac{5}{4}$
1.10	$1 + \frac{1}{2} = \frac{3}{2}$
1.11	$1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$

$E = -1$	$1.00 \cdot 10^{-1}$
$E = 0$	
$E = 1$	

$E = -1$ $\quad 1.00 \cdot 10^{-1} \quad 1.01 \cdot 10^{-1} \quad 1.10 \cdot 10^{-1} \quad 1.11 \cdot 10^{-1}$

$E = 0$ $\quad 1.00 \quad 1.01 \quad 1.10 \quad 1.11$

$E = 1$ $\quad 1.00 \cdot 10^{1} \quad 1.01 \cdot 10^{1} \quad 1.10 \cdot 10^{1} \quad 1.11 \cdot 10^{1}$

BASE 2

$E = -1$ $\quad 1/2 \quad 5/8 \quad 3/4 \quad 7/8$

$E = 0$ $\quad 1 \quad 5/4 \quad 3/2 \quad 7/4$

$E = 1$ $\quad 2 \quad 5/2 \quad 3 \quad 7/2$

BASE 10

N

Total # of $\#$ $\therefore$

$$1 + 2 \cdot (\beta - 1) \cdot \beta^{N-1} \cdot (u - L + 1)$$

$\uparrow$ sign

$\uparrow$ first digit

$$\beta = 2, N = 3,$$

Our system: $2 \cdot 2^2 \cdot (1 + 1 + 1)$

$$= 24 + 1 = 25$$

Double Precision

\# of \#

$$1 + 2 \cdot 1 \cdot 2^{5^2} \cdot 2046 = 1.9E19$$

O

Here are matlab command issued and explained during the lecture. Try these on your computer, and type "help" and command you need help with

```
x=3*((4/3)-1)-1
x=[-3:3]
y=x.^2
plot(x,y)
plot(x,y,'o')
x = linspace(-3,.5,3);

A = rand(3,3)

A(1,3)

A(:,2)

eye(4,5)

 x1 = [1:10^5];
 y1 = 1./x1;
 S1=sum(y1);
 x2 = [1:10^5:-1:1];
 y2 = 1./x2;
 S2 = sum(y2);
format long
S1-S2

x = [-5e-16:1e-18:5e-16];

y=((1-(1-x))./x);

plot(x,y,'ro')

help for

why
```